Comprehensive Exams in
Optimization

Students must take their exam in the material contained in section 1, background, and the material in either section 2 or section 3.

1. **Background:**

   (a) **Linear Algebra:** Vector spaces and subspaces, nullspaces, columnspaces, linear transformations, Gram-Schmidt orthogonalization, eigenvalues, eigenvectors, matrix diagonalization, Schur triangularization theorem, Cayley-Hamilton theorem, LU decomposition, QR decomposition, Cholesky decomposition, definite and semidefinite matrices, Schur complement, vector and matrix norms, Moore-Penrose inverse, Ritz-Rayley theorem, Courant-Fischer theorem, interlacing theorem, Sylvester law of inertia, Kronecker and Hadamard products, positive matrices and the Perron-Frobenius theorem.

   (b) **Convexity theory in $\mathbb{R}^n$:** Open and closed sets, convex sets, convex functions, convex hull, faces and extreme points, normal cones, affine independence, Caratheodory theorem, separating hyperplanes.

2. **Continuous Optimization:**

   (a) Linear programming: Basic feasible solutions, resolution or representation theorem, Separation theorem, Farkas Lemma and the theorems of the alternative, duality theorems, optimality conditions, the revised simplex algorithm, sensitivity analysis, strict complementarity theorem, parametric linear programming.

   (b) Nonlinear programming: Taylor series, Gradient, Hessian, Newton method, steepest decent method, quasi-Newton method, KKT conditions, constraint qualification conditions, quadratic programming, penalty functions.

   (c) NP-completeness, assignment problem, Hungarian method, total unimodularity, branch and bound methods.

3. **Discrete Optimization:**

   (a) Graph theory: Menger theorem, trees, planar graphs, Euler formula, graph colorings, bipartite graphs, matchings, Hall theorem, perfect graphs, chordal graphs.

   (b) Linear and nonlinear programming: Basic feasible solutions, resolution or representation theorem, Separation theorem, Farkas Lemma and the theorems of the alternative, duality theorems, the revised simplex algorithm, Newton method, steepest decent method, KKT conditions, constraint qualification conditions.

   (c) Combinatorial optimization: Shortest path problem, assignment problem, Hungarian method, max-flow min-cut theorem, min cost spanning tree problem, NP-completeness, max cut problem.

   (d) Integer programming: Total unimodularity, Birkhoff- von Nuemann Theorem, integer hull, polytopes, facets, branch and bound methods, Bender decomposition.

References:

1. **Horn and Johnson**, “Matrix analysis”.
2. **R. Webester**,“Convexity”.
3. **A. Schrijver**, “Theory of linear and integer programming”.
4. **Nemhauser and Wolsey**, “Integer and combinatorial optimization”.
5. **R.T. Rockafellar**,“Convex analysis”.
6. **Bondy and Murty**, “Graph theory”.
7. **Avriel**, “nonlinear programming, analysis and methods”.
8. **Mangasarian**, “Nonlinear programming”.
9. **Dennis and Schnabel**, “Numerical methods for unconstrained optimization and nonlinear equations”.