University of Windsor
Department of mathematics and statistics
Syllabus for the comprehensive exam in Algebra

April 7, 2023

1 Linear Algebra

1.1 Matrices

- matrix operations, inverses
- Gauss-Jordan elimination
- rank
- determinant
- Cramer's rule
- characteristic polynomial, eigenvalues, eigenvectors, eigenspaces, diagonalization, similarity
- Jordan Canonical Form, generalized eigenspaces, matrix exponentials

1.2 Vector Spaces

- vector spaces
- linear transformations, $\mathrm{Hom}(V, W)$
- bases, change of basis, dimension
- matrix of a linear transformation with respect to given input and output bases
- subspaces, quotients, dual spaces

1.3 Orthogonality

- inner products, inner product spaces
- angles, lengths, orthogonality
- projections, orthogonal complements
- Gram-Schmidt Orthogonalization Process
- bilinear form, Hermitian form
- definite, semi-definite
- symmetric, orthogonal, unitary, Hermitian, skew-symmetric, skew-Hermitian etc. operators
- adjoints

2 Abstract Algebra

2.1 Groups

- group operations, inverse
- homomorphism, isomorphism, kernels, images
- subgroups, quotients, direct products, cosets
- order
- basic well-known groups: dihedral group D_{2n}, V_4, Q_8, $\mathbb{Z}_p$, $\mathbb{Z}_n$, symmetric group S_n, alternating group A_n, $GL_n(\mathbb{K})$, $PGL_n(\mathbb{K})$, $SL_n(\mathbb{K})$
- Lagrange's Theorem
- generators and relations
- abelian, commutator, centre, centralizer, normalizer
- actions, orbit, stabilizer
- conjugation, normal subgroup, simple group, composition series, composition factors
- Isomorphism theorems
- Sylow theorems

2.2 Rings, Fields and Modules

- ring, subring, ideal, quotient
- homomorphism, isomorphism, kernels, images
- polynomial rings
- units, primes, prime ideals, maximal ideals
- principal ideal domains, Euclidean domains, Gauss Lemma, Chinese remainder theorem
- Euclidean algorithm, unique factorization
- field of quotients
- Fundamental Theorem of Field Theory, splitting fields, Fundamental theorem of Galois theory
- algebraic, transcendental
- algebraically closed, algebraic closure, extensions, degree of extension
- tensor products, exact sequences, modules over a principal ideal domain