

## THE OUTLINE FOR COMPREHENSIVE EXAMINATION IN ANALYSIS

**Set Theory and Topology:** Axiom of Choice and its equivalents, ordinals and cardinals, Schroeder-Bernstein theorem, metric spaces, neighbourhoods, bases, nets, compactness, the Tychonoff product theorem, separation axioms and countability, connectedness, the Stone-Weierstrass Theorem, the Ascoli-Arzelá Theorem, Urysohn's lemma, Tietze's extension theorem.

**Real Analysis:** Continuity, uniform continuity, differentiability of functions, convergence and absolute convergence of series, uniform convergence of functions, the inverse and implicit function theorems, Fourier series, Borel measures, Lebesgue measure and Lebesgue integral, the Lebesgue convergence theorem,  $L^p$  spaces, Fubini's theorem, Hilbert spaces, Banach spaces.

**Complex Analysis:** Analytic functions, Cauchy-Riemann equations, Cauchy's theorem, conformal mappings, the maximum modulus principle, Laurent expansions, the residue theorem and its applications, analytic continuation.

### REFERENCES

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- [3] J. Kelley, *General topology*. Graduate Texts in Mathematics, No. 27. Springer-Verlag, New York-Berlin, 1975.
- [4] J. Marsden and M. Hoffman, *Basic complex analysis*. Second edition. W. H. Freeman and Company, New York, 1987.
- [5] H. Royden, *Real analysis*. 3rd edition. Macmillan Publishing Company, New York, 1988.
- [6] W. Rudin, *Principles of mathematical analysis*. 3rd edition. International Series in Pure and Applied Mathematics. McGraw-Hill Book Co., 1976.
- [7] S. Willard, *General topology*. Dover Publications, Inc., Mineola, NY, 2004.

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